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DLF-approach as the development of Segal's chronometric theory. III: More on the tachyonic component

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Abstract

Segal's theory is outlined briefly. It is based on space-time D. There are space-times L and F, which are on equal footing with D. The main idea of the DLF-approach is that each object of nature has not just D-properties (that is, "conventional" ones) but L- and F-properties, too. The F-part of the theory is studied in more details.

1. Introduction and the world *D*

The importance of the **Biological Field** which is "a combination of different types of fields… of known and unknown origin…" has been stressed in [K-02] (p.237 and elsewhere). To try to understand those "unknown origins" we build a certain theoretical model, first. Otherwise, as someone said, "We do not have any idea how to detect those new fields, do not know what to measure". The current article is dedicated to the ***DLF*-approach** which is based on Segal's chronometric theory (see more details in [Le-03] and/or in [KL-05]). Having in mind the topic of the Conference, it is worth mentioning that significant part of paragraph I.3 of [KL-05] is dedicated to rigorous mathematical notion of **energy**. The *DLF*-theory forces to consider three types of energy instead of just one.

The chronometric theory (see surveys [Le-93], [Le-95]) has been presented in dozens of articles many of which have been published in leading mathematical, physics, and astronomy journals.

Here are some building blocks of the *DLF*-approach. Denote by M the Minkowski space-time (in its *Hermitian realization*, see Section 3 below, where the Caley map formula is given). Let D stand for the unitary group $U(2)$. The image $c(M)$ of the Caley map c (refer to [Se-76] or to [Le-95]) is a dense open subset in D.

Let us view M as a vector group. It is commutative: each left translation is the respective right translation, too. The family $\{C_y\}$ of subsets in M forms a bi-invariant cone field; each $C_y = y + C$, where C is a light cone at the origin of M. Due to the presence of the Caley map, there is the corresponding cone field on D, too. On the universal cover $D^{\sim}$ of D one can introduce *future sets* in a canonical way. These sets are determined by the above cone field and by the choice of orientation in time; they form the causal structure on $D^{\sim}$ (whereas D is *acausal* since it is compact).

Let G denote the conformal group $SU(2,2)$. Recall the well-known linear-fractional G-action on D:

$$g(z) = (Az + B)(Cz + D)^{-1} \qquad (1.1)$$

where an element g is determined by 2×2 blocks A, B, C, D. This action is canonically lifted to the $G^{\sim}$-action on $D^{\sim}$ (the latter action preserves the causal structure). Proofs of the above statements can be found in [Se-76, PaSe-82a].

Theorem 1 ([Al-76, Se-76]). *If a bijection f of* $D^{\sim}$ *preserves its causal structure then f is an element of the transformation group* $G^{\sim}$*, determined by the action* (1.1**)**.

In other words, **the geometry of such a space-time is determined by its causal structure** – a fundamentally important property!

Vector fields

$$X_0 = L_{-10},\ X_1 = L_{14} - L_{23},\ X_2 = L_{24} - L_{31},\ X_3 = L_{34} - L_{12}$$

form a left-invariant orthonormal basis on $D = U(2)$, where fifteen vector fields L_{ij} (index i is less than j, and they take on values -1, 0, 1, 2, 3, 4) are determined on D by the action (1.1). Let us keep the same symbols to denote respective vector fields on $D^{\sim}$. Globally, $D^{\sim}$ is $R^1 \times S^3$, where S^3 is represented by the group $SU(2)$. A cosmological model based on $D^{\sim}$ has been considered by many experts "to be an excellent model for the large-scale gravitational structure of the universe. It fell into disfavor only because of the belief, explicitly stated by Hubble, that there would be no redshift in it" ([Se-85, p.214]). Segal's chronometric theory explains redshift by the excess of *D*-energy over *M*-energy. That is why "there is no reason not to use the Einstein Universe $D^{\sim}$ as a gravitational model, as originally proposed". Also, see [DS-01].

In Chronometry, there is a conformal invariant R, interpreted as the radius of a three-dimensional (spherical) space. Irving Segal has put it for the (long wanted by Dirac and others) third fundamental constant – additionally to the speed of light and to the Planck's constant. If (for mathematical convenience) one takes R=1, then the scalar curvature is 6.

The conformally covariant wave operator is

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 + 1,$$

as it is shown in [PaSe-82a].

Remark 1. In the General Relativity Theory (GRT), $D^{\sim}$ is also known under the name of *Einstein static universe* (see [Kr-80, p.122]). The respective solution (of GRT Einstein equations) is interpreted as an *ideal fluid.* If not to assume R=1, then the scalar curvature is $6/(R^2)$. *Energy density* and *pressure* both equal $1/(R^2)$. *Energy conditions* hold. See [Le-07] for proofs.

2. The worlds *L* and *F*

Topologically, $L^{\sim}$ is R^4. Its relatively compact form L (being a four-dimensional orbit in D) is determined by a basis of vector fields l_1, l_2, l_3, l_4 on $U(2)$ where

$$l_1 = -(L_{-10} + L_{04} + L_{-11} + L_{14}),$$

$$l_2 = (1/2)(L_{-12} + L_{24} + 2L_{30} + 2L_{31}),$$

$$l_3 = (1/2)(L_{-13} + L_{34} + 2L_{02} + 2L_{12}),$$

$$l_4 = (1/8)(-5L_{-10} - 3L_{-11} + 3L_{04} + 5L_{14} + 4L_{23}).$$

One can prove that they generate the *oscillator* Lie algebra. The expression for the invariant form (which determines the bi-invariant metric) follows from the formula for the respective wave operator (see below).

The scalar curvature is now 0 (as shown in [Le-86b] where this world $L^{\sim}$ has been studied separately; altogether the three worlds have been studied in [Le-86a]).

Here is the expression for the conformally covariant wave operator:

$$2l_1l_4 - (l_2)^2 - (l_3)^2.$$

In terms of GRT, we now have an *isotropic electromagnetic field determined by a covariantly constant light-like vector* (see [Le-86b, p.123]). Energy conditions hold. This space-time is a special case of *plane waves.* The latter have been discussed in dozens of publications. Of special interest in our context is an article [Pe-76] by R. Penrose, where he introduced a method for taking a continuous limit of any space-time to a plane wave.

In [NaWi-93] a conformal field theory model is based on *L.* The model is an un-gauged Wess-Zumino-Witten model. In [CaJa-92], [CaJa-93] the oscillator Lie algebra *l* is used to formulate string-inspired lineal gravity as a gauge theory. However, the last two publications are seriously flawed. Namely, the upper left corner **h** (see expression (36) from [CaJa-92] and formula (3.41) from [CaJa-93]) has to be an identity matrix, rather than a diagonal matrix with 1, -1, entries. No surprise that the authors could not believe in one of their own conclusions (see their p. 249 of [CaJa-93]). In [NaWi-93] (which refers to [CaJa-92], [CaJa-93]) the invariant form in question is introduced correctly: expression (6) on p.3751.

The group *L* has been called an *oscillator* one. The *L*'s important property to admit a non-degenerate bi-invariant metric has only been noticed in early 80s: [GuLe-84], [Le-85], [MeRe-85].

From the above discussion it is clear that the building blocks *D, L* are quite well understood, and their importance (as specific worlds of the GRT and otherwise) is accepted by the physics/mathematics community. As part of the *DLF*-approach, let us now consider the **tachionic component** *F*.

$F^{\sim}$ is R^4, topologically. It is the universal cover of the Lie group *U(1,1).* Its relatively compact form *F* (being a four-dimensional orbit in *D*) is determined by an orthonormal basis of vector fields H_0, H_1, H_2, H_3 on *U(2).* Here $H_0 = L_{-10} - L_{12}$, $H_1 = - L_{-12} - L_{01}$, $H_2 = L_{02} - L_{-11}$, $H_3 = L_{34}$. These fields generate a *u(1,1),* a sub-algebra of *su(2,2).* The scalar curvature is negative 6, and

$$(H_0)^2 - (H_1)^2 - (H_2)^2 - (H_3)^2 - 1$$

is another conformally covariant wave operator.

Remark 2. Treated as the solution of Einstein equations, it is interpreted as a *tachionic fluid,* [Kr-80, p.57]. In the expression for the corresponding bi-invariant metric, there is a parameter *a* related to a choice of an invariant form on the simple *su(1,1)*-sub-algebra of *u(1,1)*. The scalar curvature is now $-6/a^2$. *Energy density* and *pressure* are both negative; $-1/(a^2)$. These statements have been proven in [Le-07]. The parameter *a* is a conformal invariant. Energy density and pressure both negative imply *energy conditions violation*, which is why the world *F* plays a special role.

Here is what M. Davidson (an expert on tachyons) writes ([Da-01, p.1]): "Tachyons captured some interest in the physics community in the 1960s and 70s [1-7] (references from [Da-01] are not included into this short article; A.L.), but they have since fallen somewhat from fashion because direct experimental evidence has not been found to support their existence, and also because of concerns about causality [8]. Arguments have been made to counter the causality objections [9], and the issue remains in dispute. There are several reasons why tachyons are still of interest today, and in fact interest may be increasing. First, many string theories have tachyons occurring as some of the particles in the theory [10], although they are generally regarded as unphysical in those theories. There are also several recent papers that assert experimental evidence that some neutrinos are tachyons [11, 12]. There is a new and extensive re-analysis of tachyon dynamics [13]. There is much discussion in the physics literature in recent years of superluminal connections implied by quantum mechanics and by the evanescent wave phenomenon of light optics as well as quantum tunneling, all indirect evidence of non-locality in nature. These and other recent developments show that tachyons are still a timely subject for investigation."

3. Pseudo-Hermitian realization of the Minkowski world *M*

Let us first recall the well-known Hermitian model for *M* (see [PaSe–82a] or [Le-95]).

Each event (or an element of *M*) is represented by a two by two Hermitian matrix *h*. The totality of all skew-Hermitian matrices *ih* forms a Lie algebra *u(2).* A typical element *(t,L,f)* of the simply connected eleven-dimensional (scaling included) Poincare group $P^{\sim}$ maps *h* into $e^tLhL^* + f$:

$$h \rightarrow e^tLhL^* + f \qquad (3.1)$$

In the above (3.1), *t* is a real number, *L* is a matrix from *SL(2,C)*, *f* is a Hermitian matrix. It is a well-known action of $P^{\sim}$.

The Caley map $c = c_D$ (which has been already mentioned in Section 1) is defined as follows:

$$c_D(h) = (1 + ih/2)(1 - ih/2)^{-1} \qquad (3.2)$$

The image of this map is an open dense subset of *U(2).* The group $P^{\sim}$ acts on *D=U(2),* too. The Caley map intertwines respective actions (see Theorem 2 of [Le-07]). The possibility of the following pseudo-Hermitian picture seems to have been unnoticed.

Recall that a two by two matrix *h* (with complex entries allowed) is in *u(1,1)* iff $h^*s + sh = 0$, where *s* is a two by two matrix *diag{1,-1}*.

Theorem 2. There is a linear bijection Q of the Lie algebra *u(2) onto u(1,1),* and there is such a $P^{\sim}$-action on *u(1,1)*, that one gets a commutative diagram (in other words, Q intertwines respective $P^{\sim}$-actions).

Proof. Choose the bijection Q, which maps a Hermitian matrix $\begin{matrix} a & b \\ c & d \end{matrix}$ into a pseudo-Hermitian matrix $\begin{matrix} a & -ic \\ -ib & d \end{matrix}$. The resulting Q is a bijection between two real four-dimensional subspaces in $\mathbf{C}^4$. If a matrix L is from SL(2,C), then it maps a pseudo-Hermitian matrix h into $A^*L'B^*hAL^TB$:

$$h \rightarrow A^*L'B^*hAL^TB, \qquad (3.3)$$

where L' is a complex conjugate of L (not a transpose), A = diag{1,i}, B = diag{-i,-1}, L^T is the transpose of L. Scaling and parallel translations both act like before, see the law (3.1). It is a straightforward exercise to verify that the two actions commute with Q.

Let us now introduce an analogue of the Caley map, C_F, from *u(1,1)* into *U(1,1):*

$$C_F\,(h) = [1 - (shs)/2][1 + (shs)/2]^{-1} \qquad (3.4)$$

Contrarily to the original Caley map C_D , its analogue C_F is *not globally defined*. As it follows from (3.4), the determinant of *[1 + (shs)/2]* vanishes on a certain (two-dimensional) hyperboloid of one sheet. That is why there is only *local* pseudo-Hermitian analogue of Theorem 2 from [Le-07]. More details are provided below.

4. F–represented *SU(2,2)*

As part of the *DLF*-approach, consider the following matrix representation of the Lie group *G = SU(2,2).* It is conjugate to the *D*–representation (the latter has been originally introduced in Segal's Chronometry; see [PaSe-82a], or [Le-95]). That conjugation is performed by the following four-by-four matrix W: W is the direct sum of -1 with a certain three by three matrix. The only non-zero entries of the latter matrix are 1s on the auxiliary diagonal. Clearly, W^2 equals the unit matrix.

The *D*–represented *SU(2,2)=G* (call it *DG,* in brief) was composed of a certain set of pseudo-unitary matrices. Overall, *DG* has been defined with the help of a distinguished diagonal matrix, *diag{1,1,-1,-1}.* Under the conjugation by W we get S= *diag {1,-1,-1,1},* which determines another copy of *SU(2,2)* (denote it by *FG*). Clearly, an isomorphism between *DG* and *FG* is carried out (via conjugation in *SL(4,C)*) by the matrix W.

The group *FG* is composed of those matrices g (with unit determinant), which satisfy

$$g^*Sg=S \qquad (4.1)$$

Similarly to the *D*–case, it is convenient to build each g of two-by-two blocks A, B, C, D. The maximal (essentially) compact subgroup K in *D*–representation consisted of block-diagonal matrices g, that is, B=C=0. There is an analogue of K in *F*-representation, call it H. Formally, H is determined by the same condition as K was. Recall that the world *F* has been defined above as the Lie group *U(1,1),* see below, equipped with a certain bi-invariant metric.

The above matrix S is the following direct sum of two-by-two matrices:

$$S = diag\{s,-s\},$$

where *s = diag{1,-1}.* Define *U(1,1)* as the totality of all two by two matrices satisfying

$$z^*sz=s \qquad (4.2)$$

Lemma (it is an analogue of Lemma 2.1.4 from [PaSe-82a]). A matrix g from *SL(4,C)* belongs to *FG* if and only if the following conditions hold

$$A^*sA - C^*sC = s,\ D^*sD - B^*sB = s,\ D^*sC - B^*sA = 0 \qquad (4.3)$$

Based on (4.1) straightforward **proof** is omitted.

Let us now introduce the following *FG*-action on ***F***: an element g maps a matrix z into $(Az + B)(Cz + D)^{-1}$:

$$gz = (Az + B)(Cz + D)^{-1} \qquad (4.4)$$

Theorem 3. Equation (4.4) defines (formally) a left action on *F=U(1,1),* that is, (g'g)z = g'(gz). If the matrix Cz+D is non-degenerate, then gz belongs to *F*.

The proof is omitted.

Remark 3. It can be shown that for an arbitrarily chosen *z* from *U(1,1),* formula (4.4) is well-defined in a certain neighborhood of *z*, and for elements *g* from a certain neighborhood of a neutral element in *FG*. Such an action is called a *local* one.

Here is an example when (4.4) is undefined. Take *z* with rows $\{2^{1/2},1\}$, $\{1, 2^{1/2}\}$; take *g* determined by blocks

$$A = D = \begin{matrix} 1 & 0 \\ 0 & cht \end{matrix}, \quad B = C = \begin{matrix} 1 & 0 \\ 0 & sht \end{matrix},$$

where $ch\, t = 2^{1/2}$, $sh\, t = -1$, values of hyperbolic cosine and of hyperbolic sine.

Theorem 4. Equation (4.4) defines a local *FG*-action on *F = U(1,1).* The subgroup H acts globally. The orbit of the neutral element (as well as the orbit of any other element of *F*) is the entire *U(1,1).*

The proof is omitted.

5. Conclusions

The main finding of the article is that the tachyonic component *F* can be introduced in a way similar to how the *D*-component has been treated in conventional physics. It is proposed that *L*- and *F*-components of an object can play the role of (long wanted) *hidden variables* of quantum mechanics.

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